

University of Saskatchewan  
Department of Electrical and Electronic Engineering



EE301 Electricity, Magnetism and Fields  
Final Examination  
Professor Robert E. Johanson

Welcome to the EE301 Final. This is a closed book and closed notes examination. A formulae sheet is attached. You may use a calculator. The examination lasts **3** hours.

Answer **six** of the seven problems. Do not answer more than six problems or severe penalties will apply.

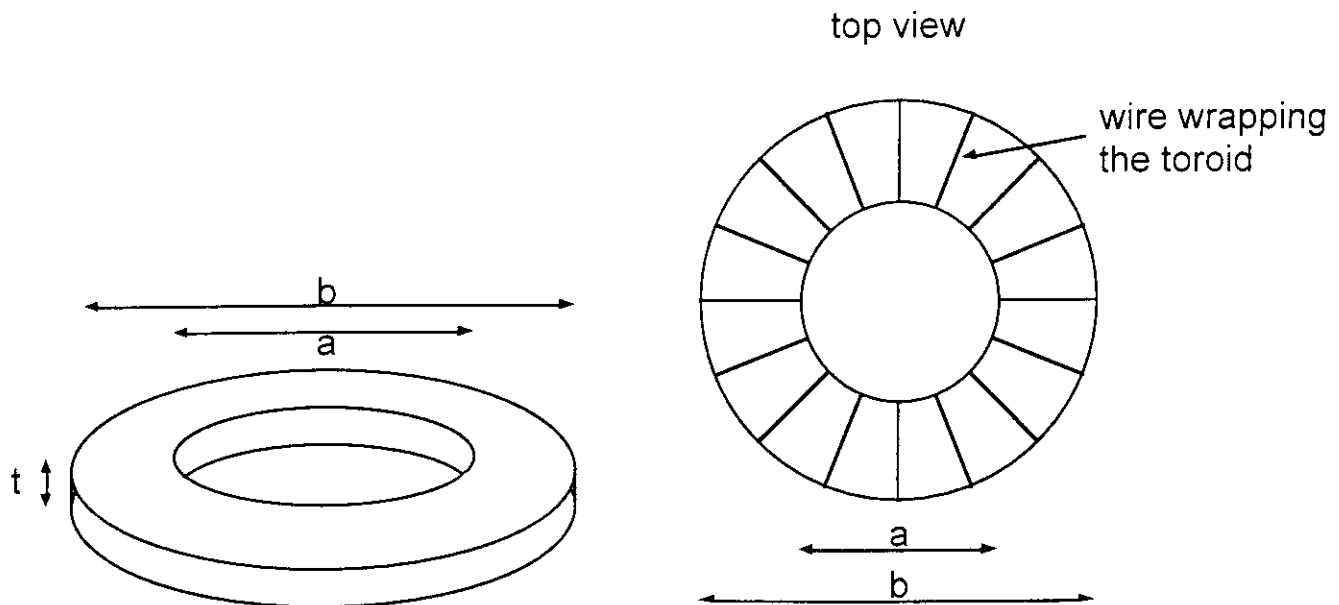
Show your work; credit will be given only if the steps leading to the answer are **clearly** shown. If a symmetry argument is used, it is sufficient to write "By symmetry we know that...". Partial credit will be given for partially correct answers but only if correct intermediate steps are shown. Each problem is weighted equally although subparts of a problem might be worth varying amounts depending on difficulty.

None of the problems require intricate mathematical manipulations. If you get stuck with an impossible integral or equation, you are likely approaching the problem incorrectly.

### Problem 1

- a) A typical thundercloud can be modeled as a parallel plate capacitor with horizontal plates of area  $10 \text{ km}^2$  and separation of  $3 \text{ km}$ . Just before a lightning bolt, the charge on the plates can be as much as  $300 \text{ C}$ . Approximately, how much energy is stored in the cloud? What is the potential difference between the top and bottom of the cloud? Why are your calculations only approximate? The dielectric constant can be taken to be 1.
- b) An isolated metal sphere of radius  $a$  is coated with a layer of dielectric with  $\epsilon_R = 3$ . The thickness of the dielectric is  $b$ . The metal is charged with total charge  $Q$ . Determine the electric field inside the metal, inside the dielectric, and outside the dielectric. You may simply write down the answer without derivation.

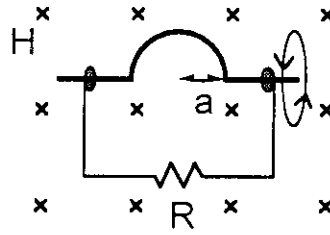
### Problem 2



A rectangular toroid is wrapped uniformly with wire creating a toroidal coil as shown. The outer diameter of the toroid is  $b$  and the inner diameter is  $a$ ; the thickness is  $t$ . The number of turns of wire is  $N$  which is very large (only a few are shown in the diagram). A current  $I$  flows through the wire.

- a) What does symmetry imply about the magnetic field  $\vec{H}(\rho, \phi, z)$  inside the coil? Ignore the discrete nature of the wire and consider the current density as continuous.
- b) Use Ampere's Law to determine the magnetic field inside the coil.

### Problem 3



In the diagram above, there is a constant magnetic field perpendicular to the plane of the paper with magnitude  $H_0$ . The circuit consists of a stationary wire bent into three sides of a rectangle and a top wire. There is a resistor  $R$  in the bottom wire. The top wire of the rectangle has a semicircular bend with radius  $a$ . The top wire is connected at each end with slip rings allowing it to turn on its axis yet retain electrical contact to the other wire. The wires have effectively no resistance. The top wire is rotated at a constant angular frequency  $\omega$ . Determine the induced voltage across the resistor.

### Problem 4

A dipole antenna broadcasts an electromagnetic wave that radiates spherically; the electric field far from the antenna is given by (in spherical coordinates)

$$\vec{E} = \frac{E_0 \omega}{cr} \sin \theta e^{j(\omega t - kr)} \vec{a}_\theta$$

- In which direction does the magnetic field point?
- Determine the equation for the magnetic field (use a Maxwell Law).
- Calculate the power density (Poynting vector) of the electromagnetic wave.
- How quickly does the power density decrease with distance from the antenna? At what angle  $\theta$  is the power density maximum?

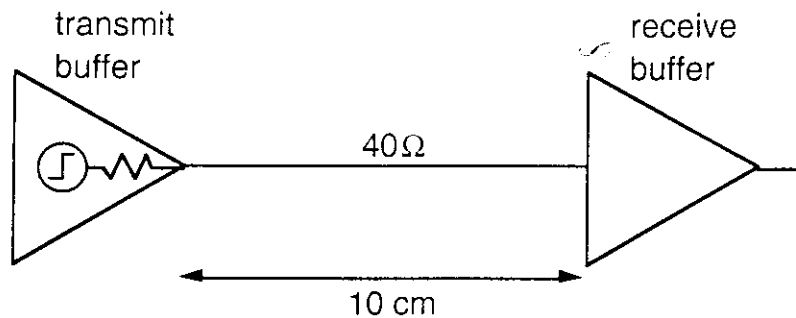
### Problem 5

- A lossless transmission line has an inductance of  $1 \mu\text{H/m}$  and a capacitance of  $100 \text{ pF/m}$ . What is the propagation constant, characteristic impedance, and the velocity of propagation?
- An antenna has an impedance of  $50 + j25 \Omega$  at  $2 \text{ GHz}$ . It is directly connected (i.e. no matching network) to a  $2 \text{ m}$  length of  $75 \Omega$  transmission line. What is the reflection coefficient off the antenna? What is the input impedance of the transmission line? The velocity of propagation in the transmission line is  $2 \times 10^8 \text{ m/s}$ .

### Problem 6

The electrical characteristics of an antenna are equivalent to a resistor of  $50\Omega$  in series with a capacitor of  $5\text{ pF}$ . The antenna is designed to broadcast at  $1.59\text{ GHz}$ . The power is delivered by a  $75\Omega$  transmission line. The velocity of propagation in the transmission line is  $2 \times 10^8\text{ m/s}$ . Design a matching scheme (you choose the type) so that no power is reflected from the antenna. Work out your matching scheme on the Smith chart provided. (Hand in the Smith chart with your booklet).

### Problem 7



The digital circuit shown above consists of a high speed transmit buffer and a high speed receive buffer connected by a  $10\text{ cm}$  long trace on a circuit board. The trace is configured to be a transmission line with a characteristic impedance of  $40\Omega$ . The output driver of the transmit buffer has an impedance of  $10\Omega$  and produces a logic high voltage of  $4\text{ V}$  and a logic low voltage of  $0\text{ V}$ . The input impedance of the receive buffer is effectively infinite. The receive buffer switches from logic low to logic high when its input exceeds  $3.5\text{ V}$ . The velocity of propagation along the line is  $1 \times 10^{10}\text{ cm/s}$ .

- Determine the reflection coefficients at each end of the line.
- Draw a bounce diagram for  $8\text{ ns}$  after the transmit buffer goes from logic low to logic high, and label each bounce line with the height of the step.
- Graph the voltage at the input of the receive buffer as a function of time for the same  $8\text{ ns}$ . Label each voltage level with the actual voltage.
- Also graph the logic state of the receive buffer over the same amount of time.
- What modification to the circuit would eliminate the obvious problem?

## Symbols and Constants

|           |                             |   |                       |
|-----------|-----------------------------|---|-----------------------|
| $F$       | force                       | $V$                                       | electric potential    |
| $Q$       | charge                      | $\vec{A}$                                 | vector potential      |
| $\vec{E}$ | electric field              | $\rho$                                    | charge density        |
| $\vec{D}$ | displacement field          | $I$                                       | current               |
| $\vec{P}$ | polarization field          | $\vec{j}$                                 | current density       |
| $\vec{H}$ | magnetic field              | $\epsilon_R$                              | relative permittivity |
| $\vec{B}$ | magnetic flux density field | $\mu_R$                                   | relative permeability |
| $\vec{M}$ | magnetization               | $\epsilon_0 \approx 8.85 \times 10^{-12}$ | F/m                   |
| $\Phi$    | magnetic flux               | $\mu_0 = 4\pi \times 10^{-7}$             | N/A <sup>2</sup>      |

## Vector Calculus

cross products:

Cartesian  $\vec{a}_x \times \vec{a}_y = \vec{a}_z \quad \vec{a}_y \times \vec{a}_z = \vec{a}_x \quad \vec{a}_z \times \vec{a}_x = \vec{a}_y$

cylindrical  $\vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z \quad \vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho \quad \vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi$

spherical  $\vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi \quad \vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r \quad \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta$

volume elements:

Cartesian  $dx dy dz$

cylindrical  $\rho d\rho d\phi dz$

spherical  $r^2 \sin \theta dr d\theta d\phi$

curl (cylindrical)

$$\vec{\nabla} \times \vec{A} = \left[ \frac{1}{\rho} \frac{\partial \vec{A}_z}{\partial \phi} - \frac{\partial \vec{A}_\phi}{\partial z} \right] \vec{a}_\rho + \left[ \frac{\partial \vec{A}_\rho}{\partial z} - \frac{\partial \vec{A}_z}{\partial \rho} \right] \vec{a}_\phi + \frac{1}{\rho} \left[ \frac{\partial(\rho \vec{A}_\phi)}{\partial \rho} - \frac{\partial \vec{A}_\rho}{\partial \phi} \right] \vec{a}_z$$

curl (spherical)

$$\vec{\nabla} \times \vec{A} = \frac{1}{r \sin \theta} \left[ \frac{\partial(A_\phi \sin \theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \vec{a}_r + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial(r A_\phi)}{\partial r} \right] \vec{a}_\theta + \frac{1}{r} \left[ \frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \vec{a}_\phi$$

## Electrostatics

Coulomb's law  $\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \vec{a}_{12}$

$\vec{E}$  of point charge  $\vec{E} = \frac{Q \vec{a}_r}{4\pi\epsilon_0 r^2}$

$\vec{E}$  of charge distribution

$$\vec{E} = \int_V \frac{\rho(\vec{r}') dV'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^2} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

Gauss's law

$$\epsilon_0 \oint_S \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{total}} = \int_V \rho dV$$

$$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}^{\text{free}} = \int_V \rho_{\text{free}} dV$$

$$\oint_S \vec{P} \cdot d\vec{S} = -Q_{\text{enclosed}}^{\text{bound}} = -\int_V \rho_{\text{bound}} dV$$

relating  $\vec{E}$  to  $V$

$$\vec{E} = -\vec{\nabla} V$$

relating  $V$  to  $\vec{E}$

$$V_{AB} = -\int_B^A \vec{E} \cdot d\vec{l}$$

$V$  of charge distribution

$$V = \int_V \frac{\rho(\vec{r}') dV'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

capacitance

$$C = Q/V$$

parallel plate capacitor

$$C = \frac{\epsilon_0 \epsilon_R A}{d} \text{ of dimensions } A \text{ and } d$$

Poisson's equation

$$\nabla^2 V = -\rho / \epsilon_0 \epsilon_R$$

Laplace's equation

$$\nabla^2 V = 0$$

linear dielectrics

$$\vec{D} = \epsilon_0 \epsilon_R \vec{E}$$

boundary conditions

$$E_T \text{ and } D_N \text{ continuous}$$

energy

$$W = \frac{1}{2} \int_V \vec{E} \cdot \vec{D} dV$$

## Magnetostatics

law of Biot-Savart

$$\vec{H} = \oint \frac{I d\vec{l} \times \vec{a}_R}{4\pi R^2}$$

law of Biot-Savart

$$\vec{H} = \int_V \frac{\vec{j} \times \vec{a}_R dV}{4\pi R^2}$$

Ampere's law

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}}$$

inductance

$$L = N\phi / I$$

vector potential

$$\vec{A} = \int_V \frac{\mu_0 \vec{j} dV}{4\pi R}$$

|                                 |   |
|---------------------------------|---|
| relating $\vec{B}$ to $\vec{A}$ | $\vec{B} = \vec{\nabla} \times \vec{A}$           |
| linear materials                | $\vec{B} = \mu_0 \mu_R \vec{H}$                   |
| boundary conditions             | $H_T$ and $B_N$ continuous                        |
| energy                          | $W = \frac{1}{2} \int_V \vec{B} \cdot \vec{H} dV$ |

### Electromagnetics

|                     |  |   |
|---------------------|--|---|
| Maxwell's equations | $\vec{\nabla} \cdot \vec{D} = \rho$                                  | $\vec{\nabla} \cdot \vec{B} = 0$  |
|                     | $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ | $\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$ |
| Faraday's law       | emf = $-d\Phi / dt$  |   |
| Poynting vector     | $\vec{P} = \text{Re}(\vec{E}) \times \text{Re}(\vec{H})$             |   |

### Transmission lines

|                          |   |
|--------------------------|---|
| propagation constant     | $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$                        |
| characteristic impedance | $Z_0 = \sqrt{(R + j\omega L)/(G + j\omega C)}$  |
| reflection coeff.        | $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$  |
| transmission coeff.      | $T = \Gamma + 1$  |
| input impedance          | $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \quad l \geq 0$     |
|                          | $Z_{in} = Z_0 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} \quad l \geq 0$ |
|                          | $\beta = 2\pi / \lambda$  |
| propagation velocity     | $v = \omega / \beta$  |
| standing wave ratio      | $\text{SWR} = (1 +  \Gamma )/(1 -  \Gamma )$  |